JEE MAINS-2019

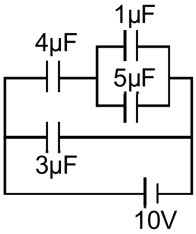
12-04-2019 Online (Evening)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above..

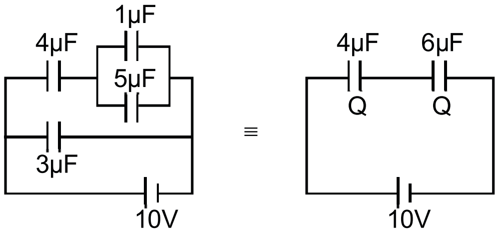
PART-A : PHYSICS

1. In the given circuit, the charge on 4 μF capacitor will be :



(1) 5.4 μC (2) 9.6 μC (3) 13.4 μC (4*) 24 μC

Sol.



$$10 = \frac{Q}{4} + \frac{Q}{6}$$

$$10 = \frac{Q}{2}\left(\frac{1}{2} + \frac{1}{3}\right) \Rightarrow 10 = \frac{Q}{2}\left(\frac{5}{6}\right) \Rightarrow Q = 24\mu C$$

2. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$(assume that the particle is at origin t = 0)

(1) $\frac{b^2\tau}{\sqrt{2}}$ (2) $b^2\tau$ (3*) $\frac{b^2\tau}{2}$ (4) $\frac{b^2\tau}{4}$

Sol. $v = b\sqrt{x}$

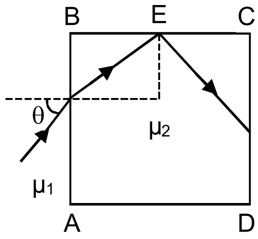
$$\frac{dx}{dt} = bx^{1/2} \Rightarrow \int_0^x x^{-1/2}dx - \int_0^\tau bdt$$

$$\Rightarrow 2\sqrt{x_\tau} = b\tau \Rightarrow \sqrt{x_\tau} = \frac{b\tau}{2}$$

$$v_\tau = b\frac{b\tau}{2} = \frac{b^2\tau}{2}$$

$$v_\tau = \frac{b^2\tau}{2}$$

3. A transparent cube of side, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 > \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



The θ must satisfy :

(1) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (2*) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (3) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (4) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

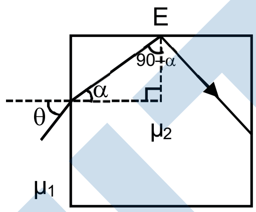
Sol. By snell's law

$\mu_1 \sin\theta = \mu_2 \sin\alpha$ ....(1)

For TIR

$\mu_2\sin(90-\alpha) > \mu_1$

From eq. (1)



$$\sin\alpha = \frac{\mu_1}{\mu_2}\sin\theta$$

$$\cos\alpha\sqrt{\frac{\mu_2^2-\mu_1^2\sin^2\theta}{\mu_2}}$$

$$\cos\alpha\sqrt{\frac{\mu_2^2}{\mu_2^2}-\frac{\mu_1^2\sin^2\theta}{\mu_2^2}}$$

$$\cos\alpha\sqrt{1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta} \quad ....(2)$$

$$\sin(90-\alpha) > \frac{\mu_1}{\mu_2}$$

$$\sin\alpha > \frac{\mu_1}{\mu_2}$$

using eq.(2)

$$\sqrt{1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta > \frac{\mu_1^2}{\mu_2^2}}$$

$$1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta > \frac{\mu_1^2}{\mu_2^2}$$

$$\frac{\mu_1^2}{\mu_2^2}\sin^2 < \left(1-\frac{\mu_1^2}{\mu_2^2}\right)$$

$$\Rightarrow \sin^2\theta < \left(\frac{\mu_1^2}{\mu_2^2}-1\right) \Rightarrow \theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)$$

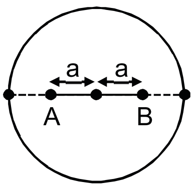
4. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a from the centre. If A and B do not experience any force, then :

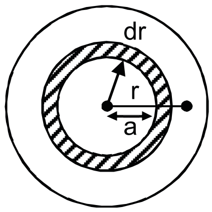
(1*) $a = 8^{-1/4}$ R (2) $a = 2^{-1/4}$ R (3) $a = \dfrac{3R}{2^{1/4}}$ (4) $a = R/\sqrt{3}$

Sol. Total charge = 2 Q

Charging density $\rho = kr$

Radius = R





d(vol.) = $4\pi r^2 dr$

dq = ρ(d(vol.))

Force on charge at B will be due to charge at A and due to force applied by the charge in sphere

$$\int dq = \int_0^R kq\, 4\pi r^2 dr$$

$$2Q = k4\pi \int_0^R r^3 dr$$

$$2Q = \frac{k4\pi R^4}{4}$$

$$k = \frac{2Q}{\pi R^4} \qquad ....(1)$$

$F_{sphere} \longleftarrow \bullet \longrightarrow F_{BA}$

$F_{BA} = F_{sphere}$

Force on charge B due to element

$$dF = \frac{k(dq)Q}{a^4} = \frac{kq(K4\pi r^3)dr}{a^2}$$

$$F = \frac{kQK4\pi}{a^4}\int_0^a r^3 dr = \frac{kQK4\pi a^2}{4}$$

$$F = kQK\pi a^2$$

$F_{BA} = F_{sphere}$

$\Rightarrow \dfrac{kQ^2}{(2a)^2} = kQ4Ka^2 \Rightarrow$ By replace value of K from (1)

$$\frac{Q^2}{4a^2} = \frac{2Q^2}{\pi R^4}\pi a^2$$

$\Rightarrow a^4 = \dfrac{R^2}{8}$

$\Rightarrow a = R\, 8^{-1/4}$

5. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62ºC, its efficiency is doubled. The temperatures of the source and the sink are, respectively.

(1) 99ºC, 37ºC (2) 124ºC, 62ºC (3*) 37ºC, 99ºC (4) 62ºC, 124ºC

Sol. $\eta = \frac{1}{6}$

$$\frac{1}{6} = 1 - \frac{T_L}{T_H} \quad ....(1)$$

$$\frac{1}{3} = 1 - \frac{(T_L - 62)}{T_H} \quad ....(2)$$

Solving eq. (1)

$$\Rightarrow \frac{1}{6} = \frac{T_H - T_L}{T_H}$$

$\Rightarrow T_H = 6T_H - 6T_L$

$6T_L = 5T_H$

$$T_H = \frac{6T_L}{5}$$

Solving eq. (2)

$$\frac{1}{3} = \frac{T_H - (T_L - 62)}{T_H} \Rightarrow T_H = 3T_H - 3T_L + 186$$

$\Rightarrow 2T_H = 3T_L - 186$

$$2 \times \frac{6T_L}{5} = 3T_L - 186$$

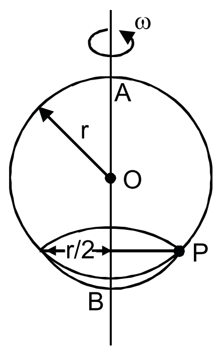
$\Rightarrow 12T_L = 15T_L - 930 \Rightarrow 3T_L = 930$

$T_L = 310$ K

$T_L = 310 - 273 = 37ºC$

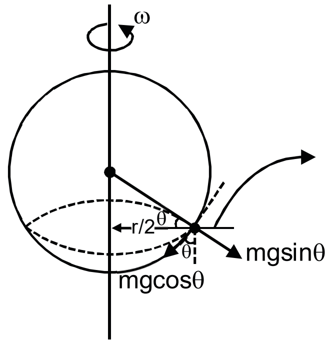
Source temp, is higher & sink temp. is lower

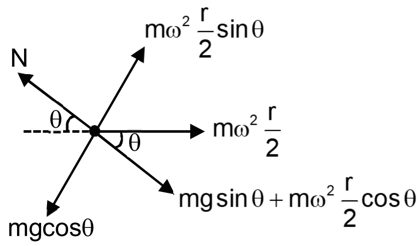
6. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to :



(1) $\left(g\sqrt{3}\right)/r$ (2) 2g/r (3) $\frac{\sqrt{3}g}{2r}$ (4*) $2g/\left(r\sqrt{3}\right)$

Sol.





$$m\omega^2\frac{r}{2}\sin\theta = mg\cos\theta$$

$$\omega^2 = \frac{2g}{r\tan\theta}$$

$$\tan\theta = \frac{\sqrt{r^2-r^2/4}}{r/2} = \frac{\frac{\sqrt{3r^2}}{4}}{\frac{r}{2}} = \sqrt{3} \Rightarrow \omega^2 = \frac{2g}{\sqrt{3}r}$$

7. A plane electromagnetic wave having a frequency v = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the Electric Field is 60 V/m. Which among the following is the acceptable magnetic field component in he electromagnetic wave ?

(1) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^{2}x+0.5\times10^{11}t)\hat{j}$

(2) $\vec{B} = 60\sin(0.5\times10^{3}x+1.5\times10^{11}t)\hat{k}$

(3) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^{3}z+1.5\times10^{11}t)\hat{i}$

(4*) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^{3}z-1.5\times10^{11}t)\hat{i}$

Sol. v = 23.9 GHz

E_0 = 60 V/m

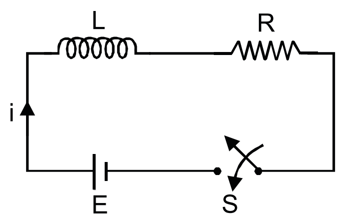
$$\because \frac{E_0}{B_0} = C \Rightarrow B_0 = \frac{E_0}{C} = \frac{60}{3\times10^{8}} = 2\times10^{-7}$$

Since the wave is propagating in positive z-direction

So acceptable magnetic field component wll be

$$\vec{B} = 2\times10^{-7}\sin(0.5\times10^{3}z-1.5\times10^{11}t)\hat{i}$$

8. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t=\frac{L}{R}$ is :



(1) $\frac{7.3EL}{R^2}$ (2) $\frac{2.7EL}{R^2}$ (3) $\frac{EL}{7.3R^2}$ (4*) $\frac{EL}{2.7R^2}$

Sol. $I = I_{max.}\left(1-e^{-\frac{Rt}{L}}\right)$ $\qquad I_{max.} = \frac{E}{R}$

$$\frac{dq}{dt} = \frac{E}{R}\left(1-e^{-R\frac{t}{L}}\right)$$

$$\int_0^{4R} dq = \frac{E}{R}\int_0^{4R}\left(1-e^{-R\frac{t}{L}}\right)dt$$

$$Q = \frac{E}{R}\left[L + \frac{L}{R}e^{-R\frac{t}{L}}\right]_0^{L/R}$$

$$= \frac{E}{R}\left[\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right]$$

$$Q = \frac{EL}{R^2 e} \Rightarrow Q = \frac{EL}{2.7R^2}$$

9. One kg of water, at 20ºC, heated n a electric fettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. the rms voltage in the mains s 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to :

(1) 10 minutes (2*) 22 minutes (3) 3 minutes (4) 16 minutes

Sol. R = 20 Ω $\qquad P = \frac{V^2}{R} = \frac{200\times200}{20} = 2000$ watt

V = 200 V

1 kg water 20ºC $\rightarrow$ 1 kg water 100ºC

Heat required $Q_1 = ms\Delta T$ = (1)(4200)(80) = 336000

1 kg water 100ºC $\rightarrow$ 1 kg vapour

Heat required $Q_2 = mL$ = (1) 2260 × 1000 = 2260000

(power) (time) = total heat required

$\Rightarrow$ 2000 × time = 2260000 + 336000

time = 1298 sec.

time = 21.63 mint. $\approx$ 22 minute

10. Consider an electron in a hydrogen atom revolving in tis second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :

(1) 6.6 Å (2) 3.5 Å (3*) 9.7 Å (4) 12.9 Å

Sol. n = 3 (second excited state)

$2\pi r_n = n\lambda_{dB}$

$$\Rightarrow \lambda_{dB} = \frac{2\pi r_3}{n} = \frac{2\times3.14\times4.65}{3} \approx 9.7Å$$

11. In an amplitude modulator circuit, the carrier wave is given by, C(t) = 4 sin(2000 πt) while modulating signal is given by, m(t) = 2sin (2000 πt). The values of modulation index and lower side band frequency are :

(1) 0.3 and 9 kHz (2) 0.5 and 10 kHz (3) 0.4 and 10 kHz (4*) 0.5 and 9 kHz

Sol. C(t) = 4 sin(20000 πt), $A_c = 4$, $\nu_c = 10000$

m(t) = 2 sin (2000 πt), $A_m = 2$, $\nu_m = 1000$

Modulation index $\mu = \frac{A_m}{A_c} = \frac{2}{4} = \frac{1}{2} = 0.5$

Lower side bond frequency = $\nu_L - \nu_m$

= 100000 – 1000

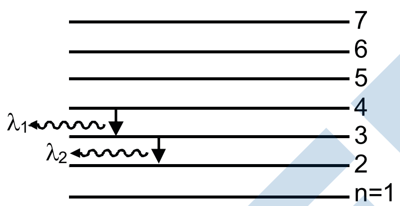
= 9000

= 9 kHz

12. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelength, λ_1/λ_2 of the photons emitted in this process is :

(1) 22/5 (2) 7/5 (3) 9/7 (4*) 20/7

Sol.



$$\frac{hc}{\lambda_1} = 13.6\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{hc}{\lambda_1} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{7}{9\times16}\right)}{\left(\frac{5}{9\times4}\right)}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{7}{9\times16} \times \frac{9\times4}{5} = \frac{\lambda_2}{\lambda_1} = \frac{7}{20}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{20}{7}$$

13. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 s crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60º to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :

(1*) 10.67 (2) 5.33 (3) 16.00 (4) 1.80

Sol. When unpolarized light of intensity I_0 passes through P_1, P_2 and P_3, let the emergent light from P_1, P_2 and P_3 and I_1, I_2, I_3. Then from Malus law

$I = I_0\cos^2\theta$l1sd

I_0 = Incident intensity

θ = Angle between pass axes and incident light

So $I_1 = \frac{I_0}{2}$ $\because \langle \cos^2\theta \rangle = \frac{1}{2}$

$$I_2 = \frac{I_0}{2}\cos^2 30^\circ = \frac{3I_0}{8}$$

$$I_3 = \frac{3I_0}{8}\cos^2 60^\circ = \frac{3I_0}{32}$$

So $I = \frac{3I_0}{32}$

$$\frac{I_0}{I} = \frac{32}{3} = 10.67$$

14. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?

(1*) 35 J (2) 40 J (3) 25 J (4) 30 J

Sol. W = 10 J at constant pressure

$W = P(V_2 - V_1)$

$= PV_2 - PV_1$

$10 = nR(T_2 - T_1) = nR\Delta T$

$\Delta Q = \Delta W + \Delta U$

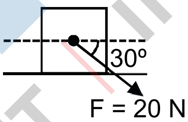
$$= 10J + \frac{nf}{2}R\Delta T$$

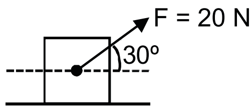
$$= 10J + \frac{5}{2}(10J)$$

$\Delta Q = 35$ J

15. A block of mass 5 kg is (i) pushed n case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in figure. The coefficient of friction between the block and floor s μ = 0.2. the difference between the acceleration of the blocks, in case (B) and case (A) will be :

($g = 10\ ms^{-2}$)

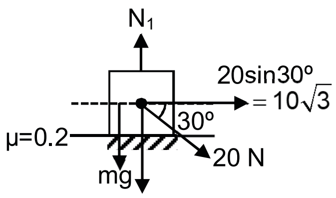




(1) 3.2 ms^{-2} (2*) 0.8 ms^{-2} (3) 0 ms^{-2} (4) 0.4 ms^{-2}

Sol.

Case-A



$20\sin30^\circ = 10$

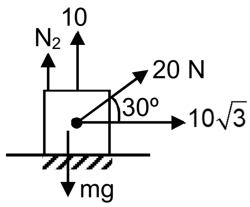
$N_1 = mg + 10$

$(f_s)_{max} = 0.2 \times (60) = 12$ N

$$a_A = \frac{10\sqrt{3}-12}{5} = \frac{17.32-12}{5}$$

$$a_A = \frac{5.32}{5}$$

Case-B



$N_2 = mg - 10$

$= 50 - 10 = 40$ n

$(f_s)_{max} = 0.2 \times 40 = 8$ N

$$a_B = \frac{10\sqrt{3}-8}{5} = \frac{17.32-8}{5}$$

$$a_B = \frac{9.32}{5}$$

difference between acceleration $a_B - a_A = \frac{1}{5}(9.32 - 5.32) = \frac{4}{5}$

$\Delta a = 0.8$ m/s^2

16. A moving col galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0($I_0 > I_g$) by connectng a shunt resistance R_A to t and (ii) into a voltmeter of range 0 to V ($V = GI_0$) by connecting a series resistance R_V to it. Then,

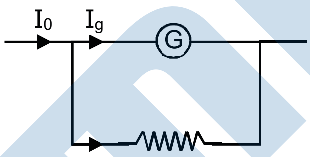
(1) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(2) $R_A R_V = G^2\left(\frac{I_g}{(I_0 - I_g)}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(3) $R_A R_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

(4*) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Sol. Galvanometer is converted into ammeter of range 0 to I_g.

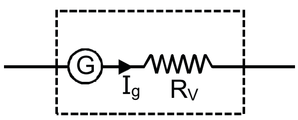


$I_0 - I_g$ $\quad$ $S = R_A$

$I_g G = (I_0 - I_g) R_A$

$$R_A = \frac{I_g G}{(I_0 - I_g)} \quad ....(1)$$

Galvanometer is converted no voltmeter of range 0 to V



$V = I_g(G + R_V)$

$GI_0 = I_g(G + R_V)$

$$R_V = \frac{G(I_0 - I_g)}{I_g} \qquad \text{....(2)}$$

So from (1) & (2)

$R_A R_V = G^2$

$$\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$$

17. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals:

(1) 1/9 (2) 1/27 (3*) 9 (4) 27

Sol. $$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow r^3 = \frac{R^3}{3^3} \Rightarrow \boxed{r = \frac{R}{3}}$$

$$V_1 = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi R^3 g}{6\pi\eta R}$$

$$V_2 = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi\left(\frac{R}{3}\right)^3 g}{6\pi\eta\left(\frac{R}{3}\right)} = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi R^3 g \times \frac{1}{27}}{6\pi\eta R \times \frac{1}{3}}$$

$$V_2 = \frac{V_1}{9}$$

18. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals :

(1) 10.0 m/s (2) 5.5 m/s (3) 15.0 m/s (4*) 2.5 m/s

Sol. $\bullet\, S_1 \qquad O \xrightarrow{u} \qquad \bullet\, S_2$

As observer goes away from source S_1 so apparent frequency

$$V_1 = \frac{(v - v_0)}{v}v \qquad v = \text{speed of round, } v_0 = \text{speed of observer}$$

$$= \left(\frac{330 - u}{330}\right) \times 600$$

$v_1 = 2 \times 330 - 2u$ ....(1)

As observer goes towards source S_2 so apparent frequency

$$v_2 = \frac{(v + v_0)}{v}v$$

$$= \left(\frac{330 - u}{330}\right) \times 600$$

$v_2 = 2 \times 330 + 2u$ ....(2)

According to question

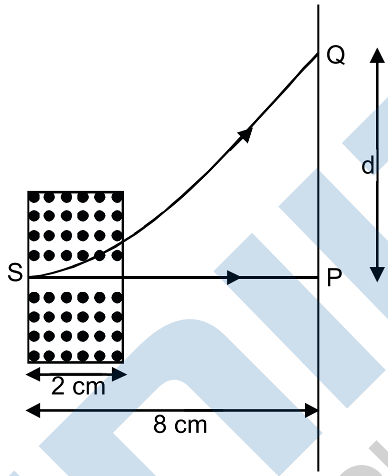
$v_2 - v_1 = 10$

$4u = 10$

$u = 2.5$ m/s

19. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} T)\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q on the screen) si :

(electron's charge = 1.6×10^{-19}C, mass of electron = 9.1×10^{-31} kg)

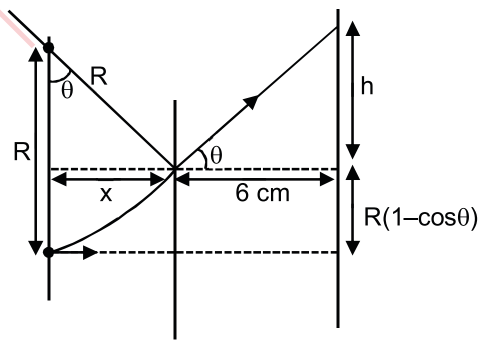


(1) 2.25 cm (2*) 12.87 cm (3) 1.22 cm (4) 11.65 cm

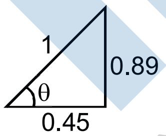
Sol. $R = \frac{\sqrt{2mk}}{qB}$

$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$

R = 2.24 cm



$\sin\theta = \frac{x}{R} = \frac{2}{2.24} = 0.89$



$\tan\theta = \frac{0.89}{0.45} \approx 1.97 = \frac{h}{6}$

h = 11.82 cm

$\cos\theta = 0.45$

$d = h + 2(1 - 0.45) = 11.82 + 2(1 - 0.45) \approx 12.9$ cm

20. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1*) 9 : 8 (2) 1 : 8 (3) 8 : 1 (4) 3 : 8

Sol. $A_1 = \frac{A_0}{2^{60/10}} = \frac{A_0}{2^6} = \frac{A_0}{64}$

$A_2 = \frac{A_0}{2^{60/20}} = \frac{A_0}{2^3} = \frac{A_0}{8}$

No. of decayed nuclei $= A'_1 = A_0 - \frac{A_0}{64} = \frac{63}{64}$

$= A'_2 = A_0 - \frac{A_0}{8} = \frac{7}{8}$

Ratio $= \frac{63}{64} \times \frac{8}{7} = \frac{9}{8}$

21. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, λ_1 = 30 cm and λ_2 = 70 cm. Then, v is equal to:

(1) 338 ms^{-1} (2*) 384 ms^{-1} (3) 379 ms^{-1} (4) 332 ms^{-1}

Sol. λ_1 = 30 cm

λ_2 = 70 cm, ν = 480

$v = 2(\lambda_2 - \lambda_1)\nu$

$= 2(70 - 30) \times 10^{-2} \times 480$

$= 2 \times 40 \times 10^{-2} \times 480$

= 384 m/sec

22. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to :

(1*) $n_0\alpha^{-3/4}$ (2) $\sqrt{n_0}\alpha^{1/2}$ (3) $n_0\alpha^{1/4}$ (4) $n_0\alpha^{-3}$

Sol. Given number density of molecules of gas as a function of r is

$n(r) = n_0 e^{-\alpha r^4}$

$\therefore$ Total number of molecule $= \int_0^\infty n(r)dV = \int_0^\infty n_0 e^{-\alpha r^4} 4\pi r^2 dr$

$\therefore$ Number of molecules is proportional to $n_0\alpha^{-3/4}$

23. A small speaker delivers 2W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? (Given reference intensity of sound as 10^{-12} W/m²)

(1) 20 cm (2) 10 cm (3*) 40 cm (4) 30 cm

Sol. $L = 10\log\frac{I}{I_0}$

$120 = 10\log\frac{I}{I_0}$

$\frac{I}{10^{-12}} = 10^{12}$

$I = 1 = \frac{P}{4\pi r^2}$

$$r^2 = \frac{2}{4\pi \times 1}$$

$$r = \sqrt{\frac{2}{4\pi}}m = 100 \times \sqrt{\frac{1}{2\pi}} = 100 \times 0.399 \approx 39.9 \approx 40cm$$

24. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to :

(1*) $3F/(\pi r^2 YT)$ (2) $6F/(\pi r^2 YT)$ (3) $F/(3\pi r^2 YT)$ (4) $9F/(\pi r^2 YT)$

Sol. $Y = \dfrac{\frac{F}{\pi r^2}}{\frac{\Delta\ell}{\ell}} \Rightarrow Y = \dfrac{F}{\pi r^2} \times \dfrac{\ell}{\Delta\ell}$

$$\Delta\ell = \frac{F\ell}{\pi r^2 Y}$$

change in length due to temperature change

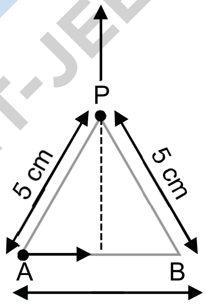
$\Delta\ell = \ell\alpha\Delta T$

$$\ell\alpha T = \frac{F\ell}{\pi r^2 Y}$$

$$\Rightarrow \alpha = \frac{F}{\pi r^2 YT}$$

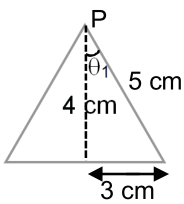
$$\boxed{Y = \frac{3F}{\pi r^2 YT}}$$

25. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5A. (see figure) ($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})



(1*) 1.5×10^{-5} T (2) 3.0×10^{-5} T (3) 2.0×10^{-5} T (4) 2.5×10^{-5} T

Sol. $B = \dfrac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$



$$= \frac{5}{4 \times 10^{-2}}\left(\frac{3}{5} + \frac{3}{5}\right) \times 10^{-7}$$

4 cm = 4×10^{-2} m

$$= \frac{5}{4} \times 2\frac{3 \times 10^{-7}}{5 \times 10^{-2}}$$

$\boxed{B = 1.5 \times 10^{-5} T}$

26. A spring whose unstretched length is ℓ has a force constant k. The spring is cut into two pieces of unstretched length ℓ_1 and ℓ_2 where, $\ell_1 = n\ell_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constant, k_1 and k_2 will be :

(1) $\frac{1}{n^2}$ (2*) $\frac{1}{n}$ (3) n^2 (4) n

Sol. $\frac{\ell_1 k}{n} = \frac{n}{\ell_1 k_2} + \frac{n}{\ell_2, k_2}$

given $\ell_1 = n\ell_2$

$k_1 = \frac{1/\ell_2}{\ell} \times k$ $\quad$ $k_2 = \frac{1/\ell_2}{\ell} \times k$

$k_1 = \frac{k}{\ell_1 \ell}$ $\quad$ $k_2 = \frac{1}{\ell\ell_2} k$

$\frac{k_1}{k_2} = \frac{\ell_2}{\ell_1} \Rightarrow \boxed{\frac{k_1}{k_2} = \frac{1}{n}}$

27. The ratio of the weights of a body on the Earth's surface to that on the surface of a planets is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ?

(Take the planets to have the same mass density)

(1) $\frac{R}{9}$ (2*) $\frac{R}{2}$ (3) $\frac{R}{3}$ (4) $\frac{R}{4}$

Sol. $\frac{W_e}{W_p} = \frac{9}{4}$ $\quad$ $M_p = \frac{1}{9} M_e$

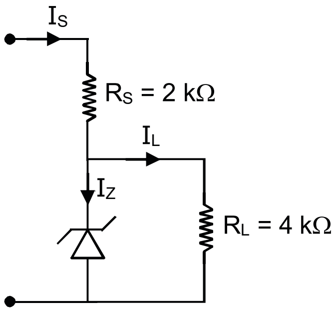
$W_e = m\frac{GM_e}{R^2}$ $\quad$ $\frac{W_e}{p} = \frac{mGM_e}{R^2} \times \frac{R^2}{mGM_p}$

$W_p = m\frac{GM_e}{R^2}$ $\quad$ $\frac{W_e}{W_p} = \frac{9R'2}{R^2}$

$\frac{9}{4} = \frac{9R'^2}{R^2} \Rightarrow R'^2 = \frac{R^2}{4}$

$\boxed{R' = \frac{R}{2}}$

28. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is maximum Zener current ?



(1*) 3.5 mA (2) 1.5 mA (3) 2.5 mA (4) 7.5 mA

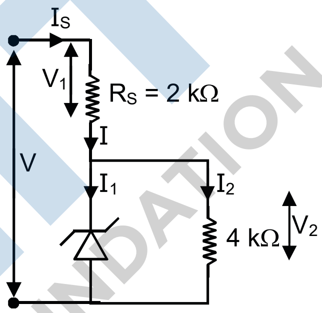
Sol. **Case - I :** V = 16 volt

V_2 = 6 V then V_1 = 10 V

$$I = \frac{10}{2k\Omega} = 5 \times 10^{-3} \text{amp.}$$

$$I_2 = \frac{6}{4 \times 1000} = 1.5 \times 10^{-3} \text{amp.}$$

$I_1 = (5 - 1.5) \times 10^{-3}$ amp.



Case – II : V = 10 volt

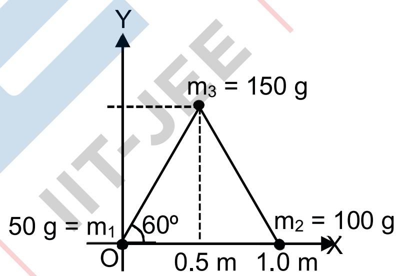
V_2 = 6 V & V_1 = 4 vol.

$$I = \frac{4}{2 \times 1000} = 2 \times 10^{-3} \text{amp.}$$

$I_1 = (2 - 1) \times 10^{-5} = 10^{-5}$ amp.

Maximum Zener current is in Case-I that is 3.5×10^{-3} amp.

29. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in figure). The (x, y) coordinate of the centre of mass will be :



(1) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$ (2) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$ (3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ (4*) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

Sol. m_1 = 50 g m_2 = 100 g m_3 = 150 g

$x_1 = 0$ x_2 = 1 m x_3 = 0.5 m

$y_1 = 0$ $y_2 = 0$ $y_3 = \frac{\sqrt{3}}{2}$

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (100)10 + (150)(0.5)}{300}$$

$$x_{COM} = \frac{100+75}{300} = \frac{175}{300} = \frac{7}{12}m$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0+(100)(0)+(150)\frac{\sqrt{3}}{2}}{300}$$

$$y_{COM} = \frac{75\sqrt{3}}{300} = \frac{3\sqrt{3}}{12} = \frac{\sqrt{3}}{4}m$$

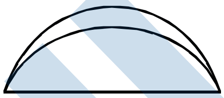
$$\boxed{(x_{COM}, y_{COM}) = \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)}$$

30. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

(1) $R^2 = h_1h_2$ (2*) $R^2 = 16\ h_1h_2$ (3) $R^2 = 4\ h_1h_2$ (4) $R^2 = 2\ h_1h_2$

Sol. Angle of projections must be

θ, $(90 - \theta)$



$$h_1 = \frac{u^2\sin^2\theta}{2g}, h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$h_1h_2 = \frac{u^4\sin^2\theta\cos^2\theta}{4g^2}$$

$$R^2 = \frac{4u^4\sin^2\theta\cos^2\theta}{g^2}$$

$$\boxed{R^2 = 16h_1h_2}$$

SECTION-B : CHEMISTRY

31. Among the following, the INCORRECT statement about colloids is :

(1) The range of diameters of colloidal particles is between 1 and to 1000 nm

(2) they are larger than small molecules and have high molar mass

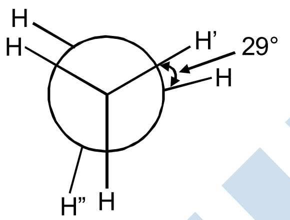
(3) They can scatter light

(4) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

Ans. [4]

Sol. Osmotic pressure of colloidal solution is lower than true solution of same concentration.

32. In the following skew conformation of ethane, H'–C–C–H" dihedral angle is :



(1) 120° (2) 58° (3) 151° (4) 149°

Ans. [4]

Sol. H'–C–C–H" = 120°

33. An 'Assertion' and a 'Reason' are given below. Choose the correct answer from the following options :

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held p-electrons, the cleavage is difficult because of strong bonding.

(1) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A)

(2) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)

(3) (A) is correct statement but (R) is a wrong statement

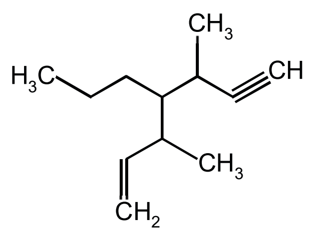
(4) Both (A) and (R) are wrong statements.

Ans. [3]

Sol. $CH_2=CH-\ddot{C}l \longleftrightarrow \overset{\ominus}{C}H_2-CH=\overset{\oplus}{C}l$

C–Cl bond is stronger due to resonance.

34. The IUPAC name for the following compound is :



(1) 3, 5-dimethyl-4-propylhept-1-en-6-yne

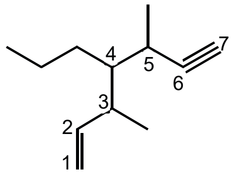
(2) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(3) 3-methyl-4-(1-methylprop-2-enyl)-1-heptene

(4) 3,5-dimethyl-4-propylhept-6-en-1-yne

Ans. [1]

Sol.



IUPCAC : – 3,5-dimethyl-4-propylhept-1-en-6-yne

35. Among the following, the energy of 2s orbital is lowest in :

(1) Li (2) H (3) Na (4) K

Ans. [4]

Sol. $E_H = \frac{-13.6Z^2}{n^2}$

As Z ↑ E_H ↓

36. The decreasing order of electrical conductivity of the following aqueous solutions is :

0.1 M Formic acid (A),

0.1 M Acetic acid (B),

0.1 M Benzoic acid (C)

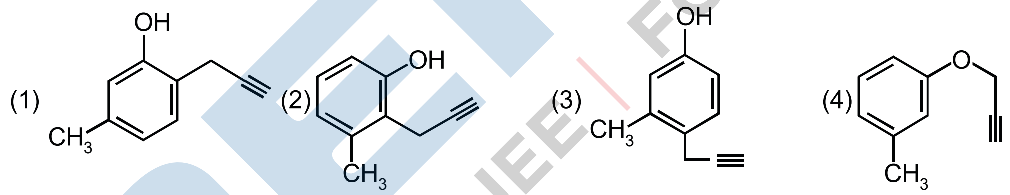
(1) A > B > C (2) A > C > B (3) C > B > A (4) C > A > B

Ans. [2]

Sol. Order of acidity : A > C > B

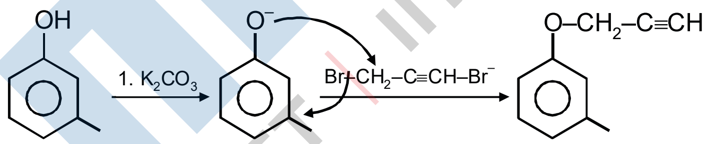
Order of electrical conductivity : A > C > B

37. What will be the major product when m-cresol is reacted with propargyl bromide ($HC\equiv C–CH_2Br$) in presence of K_2CO_3 in acetone ?



Ans. [4]

Sol.



38. The C–C bond length is maximum in :

(1) C_{60} (2) diamond (3) graphite (4) C_{70}

Ans. [2]

Sol. C–C in C_{60} = 1.4Å

C–C in C_{70} = 1.37 to 1.46 Å

Diamond = ℓ > 1.54Å

Graphite = 1.54 Å

39. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains :

(1) 18 g of carbon and 7 g of hydrogen (2) 20 g of carbon and 5 g of hydrogen

(3) 22 g of carbon and 3 g of hydrogen (4) 24 g of carbon and 1 g of hydrogen

Ans. [4]

Sol. Let hydrocarbon is C_xH_y

$$C_xH_y + O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\text{wt of carbon} = \frac{88}{44} \times 12 = 24 \text{ g}$$

$$\text{wt of hydrogn} = \frac{9}{18} \times 1 \text{ g}$$

40. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are :

(en = ethane-1, 2-diamine)

(1) 3 and 3 (2) 6 and 6 (3) 5 and 3 (4) 5 and 6

Ans. [4]

Sol. $[CoCl(en)_2]Cl$

$$\left.\begin{array}{l} \text{en} \rightarrow \text{bidentate} \\ \text{Cl} - \text{monodentate} \end{array}\right] \text{So C.N. of Co is 5.}$$

$K_3[Al(C_2O_4)_3]$

$C_2O_4^{2-} \rightarrow$ is bidentate so C–N of Al is 6

41. The primary pollutant that leads to photochemical smog is :

(1) acrolein (2) ozone (3) sulphur dioxide (4) nitrogen oxides

Ans. [4]

Sol. Photochemical smog contains oxides of nitrogen

42. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and gaseous product. mnO_2 reacts with NaCl and concentrated H_2O_4 to give a pungent gas Z. X, Y and Z, respectively, are :

(1) $KMnO_4$, K_2MnO_4 and Cl_2 (2) K_2MnO_4, $KMnO_4$ and SO_2

(3) K_3MnO_4, K_2MnO_4 and Cl_2 (4) K_2MnO_4, $KMnO_4$ and Cl_2

Ans. [1]

Sol. $$\underset{(X)}{KMnO_4} \xrightarrow{\Delta} \underset{(Y)}{K_2MnO_4} + MnO_2 + O_2$$

$$MnO_2 + NaCl + H_2SO_4 \longrightarrow MnSO_4 + \underset{(Z)}{Cl_2} + Na_2SO_4 + H_2O$$

$X = KMnO_4$, $Y = K_2MnO_4$, $Z = Cl_2$

43. The INCORRECT match in the following is :

(1) $\Delta G^0 = 0, K = 1$ (2) $\Delta G^0 < 0, K < 1$ (3) $\Delta G^0 > 0, K < 1$ (4) $\Delta G^0 < 0, K > 1$

Ans. [2]

Sol. $\Delta G = \Delta G^o + RT\ell n\theta_1$

at equil. $\Delta G = 0$

$\Delta G^o = -2.303\ RT \log K$

if $\Delta G^o < 0$

$\Rightarrow -2.303RT \log K < 0$

$\Rightarrow \log K > 0$

$\Rightarrow K > 1$

44. The INCORRECT statement is :

(1) Lithium is least reactive with water among the alkali metals

(2) LiCl crystallises from aqueous solution as $LiCl.2H_2O$

(3) Lithium is the strongest reducing agent among the alkali metals

(4) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2

Ans. [4]

Sol. $Li(NO_3)_2 \xrightarrow{\Delta} \underset{\text{(which is incorrect)}}{Li_2O + NO_2 + O_2}$

45. The pair that has similar atomic radii is :

(1) Mo and W (2) Mn and Re (3) Ti and Hf (4) Sc and Ni

Ans. [1]

Sol. Size of 3d < 4d = 5d (due to lanthanoid contraction)

So, size M_0 [illegible] W

46. The correct statement is :

(1) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate

(2) the Hall-Heroult process is used for the production of aluminium and iron

(3) pig iron is obtained from cast iron

(4) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2

Ans. [1]

Sol. $Al_2O_3 + 2NaOH + 3H_2O \xrightarrow[35-36bar]{473-523K} Na[Al(OH)_4]$

$NaOH + SiO_2 \longrightarrow Na_2SiO_3 + H_2O$

47. Which of the given statements is INCORRECT about glycogen ?

(1) It is a straight chain polymer similar to amylose (2) It is present in animal cells

(3) It is present in some yeast and fungi (4) Only α-linkages are present in the molecule

Ans. [1]

Sol. Amylose is a straight chain polymer of β-D-(+) glucose.

48. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation,

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO2 is :

(1) 2.083×10^{-3} mol L^{-1} min^{-1} (2) 8.333×10^{-3} mol L^{-1} min^{-1}

(3) 4.167×10^{-3} mol L^{-1} min^{-1} (4) 1.667×10^{-2} mol L^{-1} min^{-1}

Ans. [4]

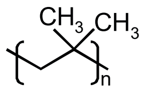
Sol. $\frac{1}{2}\frac{d[N_2O_5]}{dt} = -\frac{1}{4}\frac{d[NO_2]}{dt}$

$\Rightarrow \frac{d[NO_2]}{dt} = -2\frac{d[N_2O_5]}{dt}$

$\Rightarrow \frac{d[NO_2]}{dt} = -2\frac{(3-2.75)}{30}$

$\frac{d[NO_2]}{dt} = \frac{2\times 0.25}{30} = 1.667\times 10^{2}\ \text{mol L}^{-1}\ \text{min}^{-1}$

49. The correct name of the following polymer is :

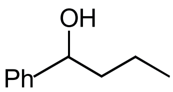


(1) Polytert-butylene (2) Polyisobutane (3) Polyisobutylene (4) Polyisoprene

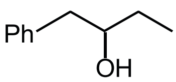
Ans. [3]

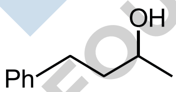
Sol. Isobutylene on polymerization will from given polymer.

50. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by NaBH4 gives Y as the major product. Y is :

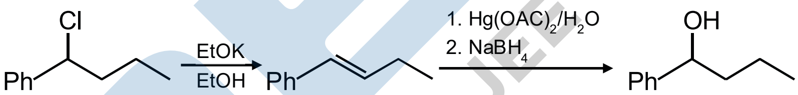
(1) 

(2) 

(3) 

(4) 

Ans. [1]

Sol. 

51. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27ºC. The osmotic pressure of the solution is : (R = 0.08206 L atm K^{-1} mol^{-1})

(1) 8.2 atm (2) 2.46 atm (3) 4.92 atm (4) 1.64 atm

Ans. [3]

Sol. $\Pi = i_1C_1RT + i_2C_2RT$

$= (i_1C_1 + i_2C_2)RT$

$= = \left[\left(\frac{0.6\times 1000}{60\times 1000}\right) + \left(\frac{1.8\times 1000}{180\times 1000}\right)\right]RT$

= (0.1 + 0.1)RT

= 0.2 RT

= 0.2 × 0.082 × 300

52. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively :

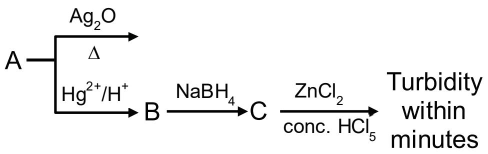
(1) 8 : 1 : 6 (2) 4 : 2 : 1 (3) 1 : 2 : 4 (4) 4 : 2 : 3

Ans. [3]

Sol. $Z_{SC} : Z_{BCC} : Z_{FCC}$

1 : 2 : 4

53. Consider the following reactions :

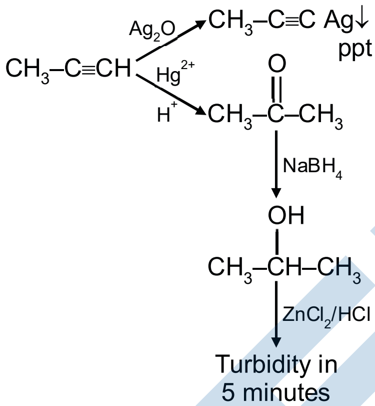


'A' is :

(1) $CH_3–C≡CH$ (2) $CH_2=CH_2$ (3) $CH_3–C≡C–CH_3$ (4) $CH≡CH$

Ans. [1]

Sol.



54. In comparison to boron, beryllium has :

(1) lesser nuclear, charge and greater first ionisation enthalpy

(2) greater nuclear charge and greater first ionisation enthalpy

(3) greater nuclear charge and lesser first ionisation enthalpy

(4) lesser nuclear charge and lesser first ionisation enthalpy

Ans. [1]

Sol. Since boron has higher nuclear charge because it has greater atomic number and lower 1^{st} I.E. then beryllium due to fully filled s-orbital.

55. The compound used in the treatment of lead poisoning is :

(1) desferrioxime B (2) Cis-platin (3) D-penicillamine (4) EDTA

Ans. [4]

Sol. (A) desferrioxime B is used for iron poisoning

(B) Cis platin is used as a anti cancer drug

(C) D-penicillamine is used for copper poisoning

(D) EDTA (ethylene diamine tetra acetate) is used for lead poisoning

56. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively, are :

(1) $Mg(HCO_3)_2$ and $MgCO_3$ (2) $Ca(HO_3)_2$ and CaO

(3) $Ca(HCO_3)_2$ and $Ca(OH)_2$ (4) $Mg(HCO_3)_2$ and $Mg(OH)_2$

Ans. [4]

Sol. $Mg(HCO_3)_2 \xrightarrow{\Delta} Mg(OH)_2 + CO_2 + H_2O$

57. The molar solubility of Cd(OH)2 is 1.84 × 10–5 M in water. The expected solubility of Cd(OH)2 in a buffer solution of pH = 12 is :

(1) 2.49 × 10–10 M (2) 1.84 × 10–9 M (3) 6.23 × 10–11 M (4) $\frac{2.49}{1.84} \times 10^{-9}$M

Ans. [1]

Sol. $[OH^-] = 10^{-2}$ for Buffer solution

$KSP = [Cd^{+2}][OH^-]^2$

$\left[Cd^{+2}\right] = \frac{K_{SP}}{\left[OH^-\right]^2} =$ solubility in buffer solution … (1)

while

$K_{SP} = 4S^3$ for $Cd(OH)_2$

$\Rightarrow K_{SP} = 4 \times (1.84 \times 10^{-5})^3$ … (2)

So solubility in Buffer solution is

$$\left[Cd^{+2}\right] = \frac{K_{SP}}{\left[OH^-\right]^2} = \frac{4 \times (1.84 \times 10^{-5})}{(10^{-2})^2} = 24.98 \times 10^{-11}$$

$[Cd^{+2}] = 24.9 \times 10^{-11}$

Solubility $= 2.49 \times 10^{-10}$

58. In which one of the following equilibria, $K_p \neq K_C$?

(1) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$ (2) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

(3) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$ (4) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

Ans. [2]

Sol. $K_p = K_C(RT)^{\Delta ng}$

$\Delta ng = 0$ so $K_p = K_C$

$\Delta ng \neq 0$ $K_p \neq K_C$

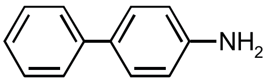
59. Which one of the following is likely to give a precipitate with $AgNO_3$ solution ?

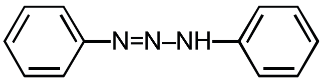
(1) $CHCl_3$ (2) $(CH_3)_3CCl$ (3) CCl_4 (4) $CH_2{=}CH{-}Cl$

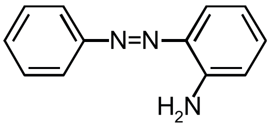
Ans. [2]

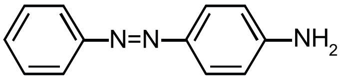
Sol. $(CH_3)_3C-Cl \xrightarrow{AgNO_3} \underset{\text{stabilises due to hyper conjugation}}{(CH_3)_3C^{\oplus}}$

60. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

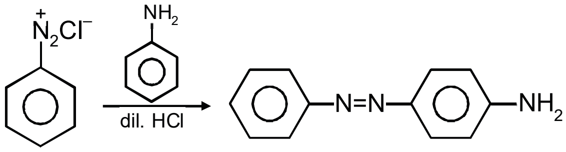
(1) 

(2) 

(3) 

(4) 

Ans. [4]

Sol. 

SECTION – C : MATHEMATICS

61. Let $a \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx$ = A(x) 2α + B(x) sin 2α + C, where C is a constant of integration then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$
(2) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$
(3) $x + \alpha$ and $\log_e |\sin(z + a)|$
(4) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$

Ans. 4

Sol. $\int \frac{\sin x \cos\alpha + \cos x \sin\alpha}{\sin x \cos\alpha - \cos x \sin\alpha} dx$

$= \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$

$= \int \frac{\sin(x-\alpha+2\alpha)}{\sin(x-\alpha)} dx$

$= \int \frac{\sin(x-\alpha)\cos 2\alpha}{\sin(x-\alpha)} dx + \int \frac{\cos(x-\alpha)\sin 2\alpha}{\sin(x-\alpha)} dx$

$= (x - \alpha) \cos 2\alpha + \sin 2\alpha \log |\sin(x - \alpha)| + C$

62. The general solution of the differential equation $(y^2 - x^3)dx - xydy = 0$ $(x \in 0)$ is :

(1) $y^2 + 2x^2 + cx^3 = 0$
(2) $y^2 + 2x^3 + cx^3 + cx^2 = 0$
(3) $y^2 - 2x^3 + cx^3 = 0$
(4) $y^2 - 2x^3 + cx^2 = 0$

Ans. 2

Sol. $(y^2 - x^3)dx - xydy = 0$ $(x \in 0)$

$y^2 - x^3 - xy\frac{dy}{dx} = 0$

$xy\frac{dy}{dx} - y^2 = -x^3$ ....(i)

Let $y^2 = v$ $2y\frac{dy}{dx} = \frac{dv}{dx}$

Put in eq^n (i)

$\frac{1}{2}\frac{dv}{dx} - \frac{1}{x}v = -x^2$

$\frac{dv}{dx} + \left(-\frac{2}{x}\right)v = 2x^2$ ...(ii)

$\text{I.F.} = e^{\int -\frac{2}{x}dx} = e^{-2\ell nnx} = \frac{1}{x^2}$

$\frac{v}{x^2} - 2x^3 - cx^2$

$y^2 = -2x^3 - cx^2$

$y^2 + 2x^3 + cx^2 = 0$

63. The equation of common tangent to the curves $y^2 = 16x$ and $xy = -4$. Is :

(1) $x - y + 4 = 0$ (2) $x + y + 4 = 0$ (3) $x - 2y + 16 = 0$ (4) $2x - y + 2 = 0$

Ans. 1

Sol. $y = mx + \frac{4}{2}$ is always tangent $y^2 = 16x$ ....(i)

If it is tangent to the $xy = -4$

$x\left(mx + \frac{4}{m}\right) = -4$

$m^2x^2 + 4x = -4m$

$m^2x^2 + 4x + 4m = 0$

for tangent $D = 0$

$16 - 16m^2 = 0 \Rightarrow m = 1$ put in eq^n (i)

$y = x + 4$

64. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

(1) $(1, -4, 1)$ (2) $(1, 4, -1)$ (3) $(2, 4, 1)$ (4) $(2, -4, 1)$

Ans. 4

Sol. $\frac{2x - y + 2z - 4}{\sqrt{2^2 + (-1)^2 + 2^2}} = \pm\left(\frac{x + 2y + 2z - 2}{\sqrt{1^2 + 2^2 + z^2}}\right)$ ...(i)

Case I : take positive sign

$2x - y + 2z = x + 2y + 2z - 2$

$x - 3y - 2 = 0$ ...(ii)

Case-II : take negative sign

$2x - y + 2z - 4 = -(x + 2y + 2z - 2)$

$2x - y + 2z - 4 = -x - 2y - 2z + 2$

$3x + y + 4z - 6 = 0$ ...(iii)

Option (4) satisfy eq^n (iii)

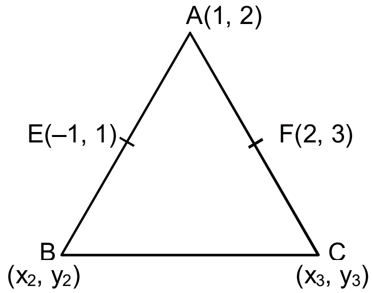
$\Rightarrow (2, -4, 1)$

65. a tringle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (22, 3). Then the centroid of this tringle is :

(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (3) $\left(1, \frac{7}{3}\right)$ (4) $\left(\frac{1}{3}, 1\right)$

Ans. 1

Sol. $\frac{x_2+1}{2} = -1, \frac{y_2+2}{2} = 1$



$x_2 = -3,\ y_2 = 0$

B(–3, 0)

$\frac{x_3+1}{2} = 2\ \&\ \frac{y_3+2}{2} = 3$

$x_3 = 3,\ y_3 = 4$

C(3, 4)

$\text{centroid}\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{2}\right)$

$\left(\frac{1-3+3}{3}, \frac{2+0+4}{3}\right) = \left(\frac{1}{3}, 2\right)$

66. The Boolean expression ~(p ⇒ (~q)) is equivalent to :

(1) $P \wedge q$ (2) $(\sim p) \Rightarrow q$ (3) $q \Rightarrow \sim p$ (4) $p \vee q$

Ans. 1

Sol. $-(p \to \sim q) = p \wedge q$

67. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true

(1) If $(A - B) \subseteq C$. then $A \subseteq C$ (2) $B \cap C \neq \phi$

(3) $(C \cup A) \cap (C \cup B) = C$ (4) If $(A - C) \subseteq B$, then $A \subseteq B$

Ans. [4]

Sol. Let A = {1, 2, 3, 4} B = {3, 4, 5, 6} C = {1, 2, 3, 4, 7, 8}

Here $A \cap B = \{3, 4\} \subseteq C$

$A - C = \phi \subseteq B$

but $A \subseteq B$

So not true (wrong) statement is 4th

If $A - C \subseteq B$ then $A \subseteq B$

68. If the are(in sq. units) bounded by the parabola $y^2 = 4\lambda x$, $\lambda > 0$, is $\frac{1}{9}$ then λ is eq

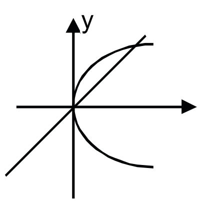
(1) $4\sqrt{3}$ (2) $2\sqrt{6}$ (3) 48 (4) 24

Ans. [4]

Sol. $y^2 = 4\lambda x\ \&\ y = \lambda x$

$\lambda^2 x^2 = 4\lambda x$

$x = 0\ \&\ x = \frac{4}{\lambda}$



$\text{Area} = \int_0^{4/\lambda}\left(\sqrt{4\lambda x} - \lambda x\right)dx = \frac{1}{9}$

$$\Rightarrow 2\sqrt{\lambda}\times\left(\frac{x^{3/2}}{\frac{3}{2}}\right)^{4/\lambda}-\lambda\left(\frac{x^2}{2}\right)_0^{4/\lambda}=\frac{1}{9}$$

$$\frac{4}{3}\sqrt{\lambda}\times\frac{(2^2)^{3/2}x}{\lambda^{3/2}}-\frac{\lambda}{2}\times\frac{16}{\lambda^2}=\frac{1}{9}$$

$$\Rightarrow \frac{32}{3\lambda}-\frac{8}{\lambda}=\frac{1}{9}$$

$$\Rightarrow \frac{32}{3\lambda}=\frac{1}{9}\Rightarrow \lambda=24$$

69. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) $\alpha\gamma$ (2) 0 (C) $\alpha\beta$ (D) $\beta\gamma$

α, β, γ are in G.P.

$\alpha x^2 + 2\beta x + \gamma = 0$ & $x^2 + x - x - 1 = 0$ have a com roots. Both roots will be common

$$\frac{\alpha}{1}=\frac{2\beta}{1}=\frac{\gamma}{-1}=\lambda$$

$$\alpha=\lambda, B=\frac{\lambda}{2}, \gamma=-\lambda$$

$$\alpha(\beta+\gamma)=\lambda\left(\frac{\lambda}{2}-\lambda\right)=\frac{-\lambda^2}{2}=\beta\gamma$$

70. If $^{20}C_1 + (2^2)\,^{20}C_2 + (3^2)^{20}C_3 + \ldots + (20^2)\,^{20}C_{20} = A(2^{\beta})$, then the ordered pair (A, β) is equal to :

(1) (420, 19) (2) (420, 18) (3) (380, 18) (4) (380, 19)

Ans. [2]

Sol. $(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + \ldots + {}^{20}C_{20}x^{20}$ .....(i)

different eq^n (i) w.r.t x

$20(1 + x)^{19} = {}^{20}C_1.1 + 2.{}^{20}C_2x + \ldots + 20\,{}^{20}C_{20}x^{19}$ ....(ii)

Multiply eq^n (ii) by x

$20x(1 + x)^{19} = {}^{20}C_1.x + 2.{}^{20}C_2x^2 + \ldots + 20\,{}^{20}C_{20}x^{20}$ ....(iii)

diff. eq^n (iii) w.r.t. x

$20[1 + x]^{19} + 19x(1 + x)^{18}] = 1.{}^{10}C_1 + 2^2.{}^{20}C_2x^{19}$ .....(iv)

put x = 1 in eq^n (iv)

$20(2^{19} + 19.2^{18}) = 1^2.{}^{20}C_1 + 2^2.{}^{20}C_2 + \ldots + (20^2)^{20}C_{20}$

$= 20 \times 2^{18}(2 + 19) = 20 \times 21 \times 2^{18} = 420 \times 2^{18}$

$A = 420,\ \beta = 18$

71. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\frac{1}{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) 3 (4) $\sqrt{3}$

Ans. [4]

Sol. Equation of plane containing both lines is

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$(x - 1)(-4 + 1) + (y - 1)(1 + 2) + z(1 + 2) = 0$

$-3(x - 1) + 3(y - 1) + 3z = 0$

$-x + 1 + y - 1 + z = 0$

$-x + y + z = 0$ distance from point (2, 1, 4) is

$$\left|\frac{-2+1+4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$$

72. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (1, 5) (2) (2, 3) (3) (3, 5) (4) (3, 10)

Ans. [4]

Sol. Equation of required circle will be

$(x - 3)^2 + (y \pm r)^2 = r^2$

$x^2 - 6x + 9 + y^2 \pm 2ry + r^2 = r^2$

$x^2 + y^2 - 6x \pm 2ry + 9 = 0$ …(i)

Length of y intercept $= 2\sqrt{f^2 - c}$ $f = \pm r$

$8 = 2\sqrt{r^2 - 9}$

$16 = r^2 - 9$

$r = 5$

So eq^n of required circle will be

$x^2 + y^2 - 6x \pm 10y + 9 = 0$

Two circles

$x^2 + y^2 - 6x + 10y + 9 = 0$ ….(ii)

$x^2 + y^2 - 6x + 10y + 9 = 0$ …(ii)

option 4th (3, 10) satisfy eq^n (iii)

73. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) $\frac{1}{4}$loss (2) $\frac{1}{2}$gain (3) $\frac{1}{2}$loss (4) 2 gain

Ans. [3]

Sol.

Win	+15	+12	–6
Prob.	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{26}{36}$

Probability of doublet = $\frac{6}{36}$

Probability of sum of 9 = $\frac{4}{36}$

Other probability = $\frac{26}{36}$

Expected gain/loss = $15\times\frac{6}{36}+12\times\frac{4}{36}-6\times\frac{26}{36}$

$$=\frac{90}{36}+\frac{48}{36}-\frac{156}{36}=\frac{-1}{2}\Rightarrow\frac{-1}{2}$$

So, $\frac{1}{2}$loss

74. Let $\alpha \in R$ and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then $S=\{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

(1) contains exactly two numbers only one of which is positive

(2) is singleton

(3) contains exactly two positive numbers

(4) is empty

Ans. [4]

Sol. $[\vec{a}\,\vec{b}\,\vec{c}]=0$

$$\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -\alpha\\ \alpha & -2 & 3\end{vmatrix}=0$$

$\alpha(3 - 2\alpha) + 1\,(-\alpha^2 - 6) + 3(-4 - \alpha) = 0$

$3\alpha - 2\alpha\,2 - \alpha^2 - 6 - 12 - 3\alpha = 0$

$-3\alpha^2 - 18 = 0$

$\alpha^2 + 6 = 0$ not possible for real α

S is empty set

75. The tangents to the curve $y = (x-2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(\frac{5}{2},-1\right)$ (2) $\left(-\frac{5}{2},-1\right)$ (3) $\left(\frac{5}{2},1\right)$ (4) $\left(-\frac{5}{2},1\right)$

Ans. [1]

Sol. $x - y - 3 = 0 \ldots$ (i) will be chord of contact of parabola

$y = x^2 - 4x + 3$

Let the required point is $P(x_1, y_1)$ chord of contact for point P is

$$\frac{y+y_1}{2} = xx_1 - 4\frac{(x+x_1)}{2} + 3$$

$y + y_1 = 2x_1x - 4x - 4x_1 + 6$

$(2x_1 - 4)x - y + (-4x_1 - y_1 + 6) = 0 \ldots$ (ii)

eq^n (i) & (ii) are same line

$$\frac{2x_1-4}{1} = \frac{-1}{-1} = \frac{-4x_1-y_1+6}{-3}$$

$\Rightarrow 2x_1 - 4 = 1$ $\quad$ $-4x1 - y_1 + 6 = -3$

$x_1 = \frac{5}{2}$ $\quad$ $-10 - y_1 + 9 = 0$

$y_1 = -1$

Ans. $\left(\frac{5}{2},-1\right)$

76. If [x] denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$, $[\cot\theta]x + y = 0$

(1) has a unique solution if $\theta \in \left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi,\frac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi,\frac{7\pi}{6}\right)$

(3) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2},\frac{2\pi}{3}\right) \cup \left(\pi,\frac{7\pi}{6}\right)$

(4) has a unique solution if $\theta \in \left(\frac{\pi}{2},\frac{2\pi}{3}\right) \cup \left(\pi,\frac{7\pi}{6}\right)$

Ans. [2]

Sol. $[\sin\theta]x + [-\cos\theta]y = 0 \ldots$ (i)

$[\cot\theta]x + y = 0 \quad \ldots$ (ii)

Case-I :

When $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ $\sin\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cos\theta \in \left(-\frac{1}{2}, 0\right) - \cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(\frac{-1}{\sqrt{3}}, 0\right)$

[sin θ] = 0, [– cos θ] = 0, [cot θ] = –1

eq^n (i) & (ii) will

$$\left.\begin{array}{l} 0x + 0y = 0 \\ -x + y = 0 \end{array}\right] \text{system will have infinitely many solution}$$

system will have infinitely many solution

Case-II :

When $\theta \in \left(\pi, \frac{7\pi}{6}\right) \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$\cos\theta \in \left(-1, \frac{\sqrt{3}}{2}\right)$

$\cos\theta \in \left(\sqrt{3}, \infty\right)$

[sin θ] = –1, [cos θ] = –1

[cot θ] = {1,2,3 ,.....}

–x – y = 0

Ix + y = 0 I = {1, 2, ...}

It will have unique solution is all cases x = 0, y = 0

77. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 24 (2) 25 (3) 27 (4) 28

Ans. [2]

Sol. Given 5 boys and n girls

total ways of forming team of 3 member under given condition

$= {}^5C_1 . {}^nC_2 + {}^5C_2 . {}^nC_1$

$\Rightarrow {}^5C_1 . {}^nC_2 + {}^5C_2 . {}^nC_1 = 1750$

$$\Rightarrow \frac{5n(n-1)}{2} + 10n = 1750$$

$$\Rightarrow \frac{n(n-1)}{2} + 2n = 350$$

$\Rightarrow n^2 + 3n = 700$

$\Rightarrow n^2 + 3n - 700 = 0$

$\Rightarrow n = 25$

78. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

(1) (2, 2) (2) (2, 2 2) (3) (2 , 2) (4) (1, 2 2)

Ans. [3]

Sol. Given 2a = 4 and 2be = 4

$\Rightarrow a = 2, be = 2$

$\Rightarrow b^2e^2 = 4$

$\Rightarrow b^2 - a^2 = 4$

$\Rightarrow b^2 = 8$

$\Rightarrow$ equation of ellipse

$$\frac{x^2}{7} + \frac{y^2}{8} = 1$$

Clearly option (3) satisfy the given curve.

79. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the robability that the candidate solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(1) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (2) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (3) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (4) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

Ans. [4]

Sol. Total problems = 50

P(Solving) = $\frac{4}{5}$

P(Not solving) = $\frac{1}{5}$

P(unable to solve less than two problems)

= P(not solving one problem) + P(not solving zero problem)

$$= {}^{50}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{50} + {}^{50}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{49}$$

$$= \frac{4^{50}}{5^{50}} + 50.\frac{4^{49}}{5.5^{49}}$$

$$= \left(\frac{4}{5}\right)^{50} + 10.\left(\frac{4}{5} + 10\right)$$

$$= \frac{54}{5}.\left(\frac{4}{5}\right)^{49}$$

80. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60º with the line x + y = 0. Then an equation of the line L is :

(1) $x+\sqrt{3}y=8$ (2) $\sqrt{3}x+y=8$

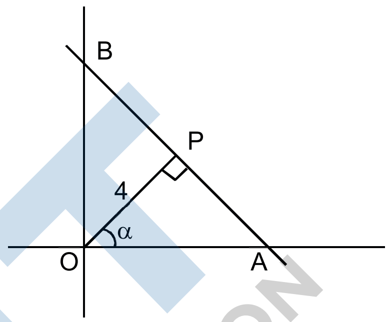
(3) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$ (4) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

Ans. [3]

Sol. OP = 4

Given OP makes 60º with x + y = 0

let slope of OP = m



$\Rightarrow \tan 60^\circ = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow \frac{m+1}{m-1}=\sqrt{3} \text{ or } -\sqrt{3}$

$\Rightarrow m+1=\sqrt{3}m-\sqrt{3} \text{ or } m+1=\sqrt{3}-\sqrt{3}m$

$\Rightarrow m\left(\sqrt{3}-1\right)=\sqrt{3}+1 \text{ or } m\left(1+\sqrt{3}\right)=\sqrt{3}-1$

$\Rightarrow m=\frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } m=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \tan\alpha=\frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } \tan\alpha=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow$ eq^n of line x cos α + y sin α = P

$\Rightarrow \left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2} \text{ or } \left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

81. A value of α such that $\int \frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$ is :

(1) 2 (2) –2 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Ans. [2]

Sol. $\int \frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$

$\Rightarrow \int_{\alpha}^{\alpha+1} \frac{(x+\alpha+1)-(x-\alpha)}{(x+\alpha)(x+\alpha+1)}dx=\log_e\left(\frac{9}{8}\right)$

$\Rightarrow \int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha}-\int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha+1}=\log_e\left(\frac{9}{8}\right)$

$\Rightarrow \log_e\left(\frac{x+\alpha}{x+\alpha+1}\right)_a^{\alpha+1}=\log_e\left(\frac{9}{8}\right)$

$$\Rightarrow \log_e\left(\frac{2\alpha+1}{2\alpha+2}\right)\left(\frac{2\alpha}{2\alpha+1}\right)=\log_e\left(\frac{9}{8}\right)$$

$$\Rightarrow \left[\left(\frac{2\alpha+1}{2\alpha+2}\right)\left(\frac{2\alpha+1}{2\alpha}\right)\right]=\log_e\frac{9}{8}$$

$$\Rightarrow \frac{(2\alpha+1)^2}{4\alpha(\alpha+2)}=\frac{9}{8}$$

$\Rightarrow 8[4\alpha^2 + 4\alpha + 1] = 9[4\alpha^2 + 4\alpha]$

$\Rightarrow 32\alpha^2 + 32\alpha + 8 = 36\alpha^2 + 36\alpha$

$\Rightarrow 4\alpha^2 + 4\alpha - 8 = 0$

$\Rightarrow \alpha^2 + \alpha - 2 = 0$

$\Rightarrow (\alpha + 2)(\alpha - 1) = 0$

$\Rightarrow \alpha = 1, -2$

82. $\lim\limits_{x\to\infty}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is :

(1) 6 (2) 1 (3) 3 (4) 2

Ans. [4]

Sol. $\lim\limits_{x\to\infty}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$

$$=\lim_{x\to\infty}\frac{x+2\sin x}{x^2+2\sin x+1-\sin^2 x+x-1}\times(\sqrt{x^2+2\sin+x+1}+\sqrt{\sin^2 x-x+1})$$

$$=\lim_{x\to\infty}\frac{x+2\sin x}{x^2+2\sin x-\sin^2 x+x}\times(2)$$

Applying L'H Rule

$$=\lim_{x\to\infty}\frac{2(1+2\cos x)}{2x+2\cos x-2\sin x\cos+1}=\frac{2(3)}{2+1}=2$$

83. The derivative of $\tan^{-1}\left(\dfrac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\dfrac{x}{2}$,where $\left(x\in\left(0,\dfrac{\pi}{2}\right)\right)$ is :

(1) $\frac{2}{3}$ (2) 1 (3) 2 (4) $\frac{1}{2}$

Ans. [3]

Sol. Given $y = \tan^{-1}\left(\dfrac{\sin x-\cos x}{\sin x+\cos x}\right)$

$$\Rightarrow y=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)$$

$$\Rightarrow y=-\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$\Rightarrow y = -\tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$$

$$\because 0 < x < \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} < -x < 0$$

$$\Rightarrow -\frac{\pi}{4} < \frac{\pi}{4} - x < 0$$

$$\Rightarrow y = -\left(\frac{\pi}{4} - x\right) \qquad \left\{\because \tan^{-1}\tan x = x \forall x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right.$$

$$\frac{dy}{d(x/2)} = \frac{1}{(1/2)0} = 2$$

84. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :

(1) [2, 6] (2) [3, 7] (3) [1, 4] (4) R

Ans. [1]

Sol. Given $\cos 2x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$$

$$\Rightarrow \sin x = \frac{\alpha \pm (\alpha - 8)}{4}$$

$$\Rightarrow \sin x = \frac{\alpha + \alpha - 8}{4}, \frac{\alpha - \alpha + 8}{4}$$

Sin x =2 (Not possible)

For solution

$$-1 \le \frac{2\alpha - 8}{4} \le 1$$

$\Rightarrow -4 \le 2\alpha \le 12$

$\Rightarrow a \in [2, 6]$

85. Let $z \in C$ with Im(z) = 10 and it satisfies $\frac{2z - n}{2z + n} = 2i - 1$ for some natural number n. Then :

(1) n = 20 and Re(z) = –10 (2) n = 40 and Re(z) = 10

(3) n = 40 and Re(z) = –10 (4) n = 20 and Re(z) = 10

Ans. [3]

Sol. Let z = x + 10i

Given $\frac{2z-n}{2z+n}=2i-1$

$\Rightarrow \frac{2(x+10i)-n}{2(x+10i)+n}=2i-1$

$\Rightarrow$ (2x – n) + 20i = (2i – 1) [(2x + n) + 20i]

Comparing real and imaginary part

$\Rightarrow$ 2x – n = 2(–20) – (2x + n) and 20 = 2(2x + n) – 20

$\Rightarrow$ 2x – n = –40 – 2x – n and 20 = 4x + 2n – 20

$\Rightarrow$ 4x = –40 and 4x + 2n = 40

$\Rightarrow$ x = –10 and –40 + 2n = 40

$\Rightarrow$ n = + 40

$\Rightarrow$ n = 40 and Re(z) = –10

86. Let f(x) = 5 – |x – 2| and g(x) = |x + 1|, x $\in$ R. If f(x) attains minimum value at β, then

$\lim_{x\to\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{3}{2}$ (4) $-\frac{3}{2}$

Ans. [1]

Sol. f(x) = 5 – |x – 2|

f(x) attains maximum value when

|x – 2| = 0 $\Rightarrow$ x = 2 = α

g(x) = |x + 1|

g(x) attains minimum value of x = –1 = β

$$\lim_{x\to\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

$$=\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$

$$=\frac{(2-1)(2-3)}{(2-4)}=\frac{1}{2}$$

87. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:

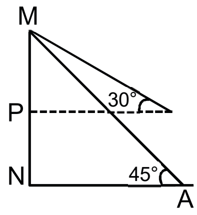
(1) 15(1 + 3) (2) 15(3 – 3) (3) 15(3 + 3) (4) 15(5 – 3)

Ans. [3]

Sol. AB = 30 m = NP

In ΔANM

$\tan 45° = \frac{MN}{AN} = 1$



$\Rightarrow$ MN = AN

PM = MN – 30

= AN – 30

In ΔBPM

$\tan 30° = \frac{PM}{PB} = \frac{AN-30}{AN}$

$\frac{1}{\sqrt{3}} = \frac{AN-30}{AN}$

$AN = \sqrt{3}AN - 30\sqrt{3}$

$AN = \frac{30\sqrt{3}}{\sqrt{3}-1} = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 15(3+\sqrt{3})$

88. A value of $\theta \in \left(0, \frac{\pi}{3}\right)$, for which $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is :

(1) $\frac{\pi}{18}$ (2) $\frac{\pi}{9}$ (3) $\frac{7\pi}{24}$ (4) $\frac{7\pi}{36}$

Ans. [2]

Sol. $\theta \in \left(0, \frac{\pi}{3}\right)$

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Expanding along first column

$\Rightarrow 2[1 - 0] - 1[-4\cos 6\theta] = 0$

$\Rightarrow 2 + 4\cos 6\theta = 0$

$\Rightarrow \cos 6\theta = -\frac{1}{2}$

$\Rightarrow 6\theta = \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$

89. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 120 (2) 150 (3) 280 (4) 200

Ans. [4]

Sol. $a_1, a_2, a_3, ..., a_n$ are in A.P.

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40$

$\Rightarrow a + 7d = \frac{40}{3}$

$S_{15} = \frac{15}{2}[2a + 14d]$

$= 15[a + 7d]$

$= 15 \times \frac{40}{3}$

$= 200$

90. The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right).\left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to :

(1) –36 (2) –108 (3) 36 (4) –72

Ans. [1]

Sol. $\left(\frac{1}{60} - \frac{x^8}{81}\right).\left(2x^2 - \frac{3}{x^2}\right)^6$

term independent of x will be

$\frac{1}{60}$ × term independent of x in $\left(2x^2 - \frac{3}{x^2}\right) - \frac{1}{8} \times$ term of z^{-8} in $\left(2x^2 - \frac{3}{x^2}\right)^6$

T_{r+1} in $\left(2x^2 - \frac{3}{x^2}\right)^6$ will be

$T_{r+1}\text{in} = {}^6C_r \left(2x^2\right)^{6-r}\left(-\frac{3}{x^2}\right)^r$

$= {}^6C_r\, 2^{6-r}\, (-1)^r \times 3^r \times x^{12-2r-2r}$

Case-I :

For term independent of x is $12 - 4r = 0 \Rightarrow r = 3$

$T_4 = -\,{}^6C_3 \times 2^3 \times 3^3 x^6 = -20 \times 2^3 \times 3^3$

Case-II :

For term of x^{-8} $12 - 4r = -8 \Rightarrow 4r = 20 \Rightarrow r = 5$

$T^6 = {}^6C_5 .\, 2^1 .\, (-1).\, 3^5 .\, x^{-8}$

Required ans. $= \frac{1}{60} \times (-20) 2^3 \times 3^3 - \frac{1}{81} \times 6 \times 2(-1) \times 3^5$

$= -72 + 36 = -36$